Research Report: 2008 (P18703) - Florian Sobieczky

The following is an overview of the research activity within the FWF-project P18703 ('Random Walks on Random Subgraphs of Transitive Graphs') in 2008:

There are two main themes (see also the original research statement): **a.)** Return probabilities and **b.)** phase transitions of random walks. For both of these sub-topics, the aim was to consider random subgraphs of transitive graphs as state spaces of the random walks, typical examples being percolation. The reason why transitive 'host'-graphs for the random subgraphs were chosen as the specific object of investigation lies in their prominence with respect to the possible symmetries the random graph valued processes may have in this case: an *invariant* percolation carries its name from the invariance of the associated probability measure with respect to transformations from a group of automorphisms of the underlying host-graph.

a.) The motivating observation for project P18703 was a result on the upper bound of the expected return probability: assuming unimodularity of the transitive graph, there is another stricter bound compared with the only general one available if transitivity is not assumed. This result is obtained by an interlacing technique and written down in []. The first part of the program of project P18703 was to investigate whether further results concerning the estimation of the return probability could be obtained using interlacing. The results are described in section 1.).

b.) A second observation made at the beginning time of the project [4] (and falling in the field of 'Changes/invariance of graph properties under random perturbations') concerns the amenability of a graph: During a discussion with Sara Brofferio, the author learned about the amenability of the Cayley graph of the lamplighter group. The question of its stability under the random perturbation given by Bernoulli percolation had already been investigated in a paper by Chen and Peres (3). As this transitive graph is in a direct way related to so called horocyclic products of trees (1), the aim was to extend the results about stability of amenability under random perturbations to percolations on the trees of the horocyclic product, rather than percolation on the horocyclic product itself. This lead to three papers dealing with horocyclic products of three different types of trees - see section 2.).

1.) Return probability bounds for random walks using eigenvalue comparison techniques

1.1) An interlacing technique for spectra of random walks and its application to finite percolation clusters [1]

Given a percolation on a graph with bounded geometry, the expected return probability is bounded from above in the regime of finite percolation clusters. It is shown, that if the graph ist transitive, and unimodular (i.e. has a transitive, unimodular sub-group of the

automorphism group), and the percolation is invariant (with respect to this sub-group), there is a downscaling of the upper bound in comparison with the general case. This is an example where the additional information given by invariance of the percolation leads to an increase in precision of an estimate regarding the return-probability.

Compared to its original form, several improvements have been added to the paper. (Submitted to JOTP).

1.2) Bounds for the return probability of the delayed random walk on finite percolation clusters in the critical case [2]

The special property of a finite graph to be Hamiltonian can be used to compare (estimate) the return probability of a continuous time random walk (CTRW) on the graph with the return probability of the same type of CTRW on a cycle of equal order (size of vertex-set). Although most finite graphs are not Hamiltonian, the cartesian product of it with a cycle of suitable length is (as shown in (2)). Since the spectrum of cartesian products of graphs decompose into the spectra of its components in a well-known way, this relation can be used to estimate the return probability of a CTRW on any finite, connected, undirected graph. The paper uses this to obtain an estimate of the expected return probability of CTRW on the finite clusters of critical percolation. Examples include percolation on $\mathbb{Z}^2$ and on regular trees. This paper has been submitted to Random Structures and Algorithms.

1.3) Comparison techniques for random walks on infinite graphs [3]

The techniques developed in the former two papers are extended here to infinite graphs by applying them to large 'squares' with time-dependent size. A condition on the validity of a relation between the return probability and the normalised trace is derived. For the class of percolation graphs which fulfill this condition, bounds for the expected return probability are given. This work exists only as a preprint, so far.

2.) Amenability of horocyclic products

2.1) Amenability of horocyclic products of percolation-trees [5]

The horocyclic product of two infinite exponentially growing trees is a graph of exponential growth. Well-known examples of such graphs involving regular trees are known to be amenable. The paper shows the stability of amenability under certain random perturbations involving percolation. Namely, the horocyclic product of percolation sub-trees of regular trees are considered. Amenability is proven, and a strong form of amenability (related to 'anchored expansion') is shown to prevail under a symmetry condition of the two percolation trees involved. This paper has been submitted to 'Markov processes and related fields'.

2.2) Amenability of horocyclic products of Galton-Watson trees [6]
(joint work with Prof. Vadim Kaimanovich)

The results of the former paper motivated the joint work during several visits of Prof. Kaimanvoch to Graz and during a visit of the author of the Erwin Schroedinger Institut, where a workshop about amenability was held (organised partly by Prof. Kaimanovich). In this paper, by use of the theory of 'treed' equivalence relations (in extension to the theory of measurable equivalence relations as develeoped by Feldman and Moore), results about the amenability of horocyclic products are found, which hold under a similar condition on the growth of random trees, as was found in the former paper. In particular, they imply the existence of an invariant measure on the equivalence relation. Under these circumstances, amenability has been shown preciously by Prof. Kaimanovich in 1997. Two papers are planned. The first one is ready an will appear in the proceedings of .... The second one exists only as a preprint, and has additional results on the rate of escape of the simple random walk on the horocyclic product. They show the existence of amenable graphs with positive 'speed'. This paper has been submitted to the Proceedings of 'Probabilistic Approach to Geometry', held from July 28 until August 8, 2008.

2.3) Amenability of horocyclic products of uniformly growing trees [7]
(joint work with Prof. Daniel Lenz and Dr. Ivan Veselić)

Again, it is asked under what conditions the horocyclic product of two trees is amenable. It is shown, that given uniform growth of the trees, amenability prevails if and only if the symmetry condition of the two former papers on the growth of the trees is fulfilled. Examples of such uniformly growing trees include substitution trees, and trees with finitely many cone-types. The result shows a distinct difference in comparison with Galton-Watson trees (and therefore also percolation sub-trees) where non-amenability only appears in horocyclic products of trees with strict locally differing growth. The paper is still in preprint form.

Bibliography of own papers:

1. F. Sobieczky: 'An interlacing technique for spectra of random walks and its application to finite percolation clusters', 3/2008, arXiv:math/0504518v4, subm. to: JOTP

2. F. Sobieczky: 'Bounds for the return probability of the delayed random walk on finite percolation clusters in the critical case', 1/2009, arXiv:0812.0117v2, subm. to: RSA

3. F. Sobieczky: 'Comparison techniques for random walks on infinite graph', 6/2008, preprint

4. F. Sobieczky: 'Anchored isoperimetry of products of trees', 9th Vilnius Conference on Probability Theory, 2006 Abstracts, p. 296

5. F. Sobieczky: 'Amenability of horocyclic products of percolation trees ', 3/2009, arXiv:0903.3140, subm. to: MPRF

6. V.Kaimanovich, F. Sobieczky: 'Stochastic homogenization of horospheric products of trees', Proceedings of the conference about 'Probabilistic Approach to Geometry' organized by the Mathematical Society of Japan, 3/2009

7. D. Lenz, F. Sobieczky, I. Veselić: 'Amenability of horocyclic products of uniformly growing trees', preprint

Bibliography of other papers:

1. L.Bartholdi, W.Woess:'Spectral computations on lamplighter groups and Diestel-Leader graphs', J. Fourier Analysis Appl., 11, 2, 175–202, 2005

2. V. Batagelj, T. Pisanski: 'Hamiltonian cycles in the cartesian product of a tree and a cycle', Discr. Math,. 38, 311-312, 1982

3. D. Chen, Y. Peres, G. Pete: 'Anchored expansion, percolation and speed', Ann. of Probab. 32, **no.** 4, (2004), pp. 2978-2995,

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5. T. Nagnibeda, W. Woess: 'Random Walks on Trees with Finitely Many Cone Types', Journ. of Theor. Probab. 15, No.2, (2002), pp. 383-422