Risk theory and risk management in actuarial science
Winter term 2018/19

6th work sheet

41. Consider a default-free 0-coupon bond with maturity T and face value 1 as well as a risky counterpart with the same maturity and the same face value. Assume that the market of these bonds is arbitrage free and let $\mathbb{Q}$ be a risk-neutral probability measure such that the discounted prices of the bonds are martingales with respect to $\mathbb{Q}$. Let $Q_T := \mathbb{Q}(\tau < T)$ be the risk-neutral probability of default before T.

    (a) Assume that the recovery rate $1-\delta$ is constant and that $1-\delta = 0$. Assume moreover that the interest rate is a constant r. Derive a relationship between Q_T and the prices $p_0(0,T)$ and $p_1(0,T)$ of the default-free bond and the risky bond, at time $t = 0$, respectively.

    (b) Let $Y_i(0,T) := -\frac{\ln p_i(0,T)}{T}$ be the yield of the default-free bond and the yield of the risky bond, for $i \in \{1,2\}$, respectively. Let $S_1(0,t) := Y_1(0,t) - Y_0(0,t)$ be the risky bond's yield spread. Write the relationship obtained in (a) in terms of $S_1(0,t)$.

    (c) Consider a risky 0-coupon bond B_1 with maturity 5 years and yield spread 1.3% and a defaultable 0-coupon bond B_2 with maturity 10 years and yield spread 1.7%, both issued by the same company. Estimate the risk-neutral probability that B_2 defaults before maturity but 5 years after its emission.

42. (a) Consider a risky 0-coupon bond with face value 1, 5 years maturity and a yield spread of 0.5%. Assume that according to data provided by some rating agency the historical 5 year default probability of this bond is $P_T := \mathbb{P}(\tau < 5) = 0.57\%$. Denote by R_1 the objective (and constant) discount rate appropriate for this risky bond. Compute Q_T and show that $R_1 - r = \frac{1}{T}\ln\left(\frac{1-P_T}{1-Q_T}\right)$ holds, where r and Q_T have the same meaning as in Exercise 41. How would you interpret $R_1 - r$ from the point of view of an investor (i.e. bond holder)?

    (b) Consider again the situation as in (a) with the only modification that the recovery rate is $1-\delta = 0.5$ in the *recovery of face* model. Compute Q_T in this case, compare it to the value of Q_T obtained in (a) and give an interpretation of the results.