Risk theory and risk management in actuarial science
Winter term 2018/19

2nd work sheet

13. Show that the following distributions belong to $MDA(\Phi_\alpha)$:
    (a) Cauchy: $f(x) = (\pi(1+x^2))^{-1}$, $x \in \mathbb{R}$.
    (b) Student: $f(x) = \frac{\Gamma((\alpha+1)/2)}{\sqrt{\alpha\pi}\Gamma(\alpha/2)(1+x^2/\alpha)^{(\alpha+1)/2}}$, $\alpha \in \mathbb{N}$, $x \in \mathbb{R}$

14. Show that The following distributions belong to $MDA(\Lambda)$:
    (a) Normal: $F(x) = (2\pi)^{-1/2}\exp\{-x^2/2\}$, $x \in \mathbb{R}$.
    (b) Exponential: $f(x) = \lambda^{-1}\exp\{-\lambda x\}$, $x > 0$, $\lambda > 0$.
    (c) Lognormal: $f(x) = (2\pi x^2)^{-1/2}\exp\{-(\ln x)^2/2\}$, $x > 0$.

15. (a) Let X be a random variable with distribution function F. Derive an equation relating $VaR_\alpha(X)$, $CVaR_\alpha(X)$ and $e_X(q_\alpha)$, where $q_\alpha := VaR_\alpha(X)$ and $\alpha \in (0,1)$.
    (b) Compute the mean excess function of the exponential distribution, i.e. compute $e_X(u)$ for $X \sim Exp(\lambda)$ and $u \geq 0$.
    (c) Use the result form (a) to compute $CVaR_\alpha(X)$ in the case of the exponential distribution, i.e. for $X \sim Exp(\lambda)$, $\alpha \in (0,1)$.

16. Consider a random variable X with distribution function X for which the approximation $\bar{F}_u(x) \approx \bar{G}_{\gamma,0,\beta(u)}(x)$ holds with $\gamma \notin \{0,1\}$. Show that this implies the approximation $\bar{F}_v(x) \approx \bar{G}_{\gamma,0,\beta(u)+\gamma(v-u)}(x)$, $\forall v \geq u$. Use this fact and the formula for the mean excess function $e_{G_{\gamma,0,\beta}}(x)$ of the generalied Pareto distribution (cf. the lecture) to show that $e_X(v)$ is linear in v for $v \geq u$ for any fixed threshold $u > 0$.

17. Consider a portfolio consisting of the following index futures *Dow Jones Industrial Average (^DJI), S&P 500 ETF (^GSPC), Nasdaq Composite (^IXIC), DAX (^GDAXI)* and *ATX (^ATX)* with one piece per each index future. Estimate value at risk $\mathrm{VaR}_{0.90}$ of the weekly losses of this portfolio in two ways as described below. Then compare and comment on the obtained results.
    (a) Use historical simulation of the losses over the last 10 years, from October 27, 2008 until October 26, 2018.
    (b) Use the variance-covariance method, again based on the weekly logarithmic returns of the last 10 years as in (a).

    The data can be downloaded from `finance.yahoo.com`: search for the required index future (you can well search for the abbreviations given in paranthesis above), click 'Historical Prices', update the 'Time Period' and 'Frequency' appropriately, and finally klick on 'Download data'. Use the adjusted close prices to compute the weakly logarithmic returns of the single index futures.

18. Use the bootstrapping method to compute a centered confidence interval with confidence level 10% for the $\mathrm{VaR}_{0.95}$ and $\mathrm{CVaR}_{0.95}$ of the weakly loss of a portfolio given as described in Exercise 17. Experiment with different values of the parameter N which specifies the number of repetitions of bootstrapping while calculating one estimator per repetition.

19. Consider the daily logarithmic returns (based on adjusted close prices) of the BMW and Siemens assets, *BMW.F* and *SIA.AS*, respectively, over the time interval October 27, 2008, and October 26, 2018. Use the Hill estimator to get an approximation of the coefficient of the regular variation and determine the corresponding estimates for $\mathrm{VaR}_{0.90}$ and $\mathrm{CVaR}_{0.90}$ for each of these assets. Specify a plausible range of values of k to be chosen (depending on the sample size) and generate the Hill plot for those values of k. Based on the Hill plot make a suggestion for an appropriate value of k to be used and argue your choice carefully. Use `yahoo.finance.com` as a data source (see Exercise 17).

20. By means of the qq-plot check whether a normal distribution or a heavy tailed distribution like the (generalized) Pareto distribution is more appropriate to model the right tail of the loss distribution of the BMW and Siemens assets as described in Exercise 19, respectively. To this end you should compare the empirical quantiles of the above mentioned losses to the (numerically or analytically) computed quantiles of the reference distributions (i.e. a normal and a generalized Pareto distribution) and summarize the results graphically as described schematically in the lecture.