Risk theory and risk management in actuarial science
Winter term 2017/18

7th work sheet

38. Apply the Poisson mixture model involved in CreditRisk$^+$ (cf. lecture) in the case of a credit portfolio with $n = 100$ credits amd m risk factors, where $m = 1$ or $m = 5$, and $\bar{\lambda}_i = \bar{\lambda} = 0.15$ for $i = 1, 2, \dots, n$, $\alpha_j = \alpha = 1$, $\beta_j = \beta = 1$, $a_{i,j} = 1/m$, for $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$.

    (a) Use the probability generating function $g_N(t)$ for the number of losses (as derived in the lecture) to compute the probability that k creditors will default, for $k = 1, 2, 3$, and for both cases $m = 1$ and $m = 5$.

    (b) Show that for $k > 1$ and arbitrary $m \in \mathbb{N}$ the derivatives of the probability generating function $g_N(t)$ cane be computed by the following recursive formula

    $$g_N^{(k)}(0) = \sum_{l=0}^{k-1} \binom{k-1}{l} g_N^{(k-1-l)}(0) \sum_{j=1}^{m} l! \alpha_j \delta_j^{l+1} ,$$

    where δ_j are as derived in the lecture, $j = 1, 2, \dots, m$.

    (c) Implement the above recursion in a software of your choice in order to compute the probability that k creditors will default, for $k = 1, 2, \dots, 100$, and for both cases $m = 1$ and $m = 5$. Vizualize the computed default probabilities graphically for both cases and try to interprete the results.

39. A bank has a loan portfolio of 100 loans. Let X_k be the default indicator for loan k such that $X_k = 1$ in case of default and 0 otherwise, for $k \in \{1, \dots, 100\}$.

    (a) Supoose that X_k are independent and identically distributed with $P(X_k = 1) = 0.01$. Compute the expected value $E(N)$ of the number N of defaults and $P(N = k)$ for $k \in \{0, 1, \dots, 100\}$.

    (b) Consider the risk factor Z which reflects the state of the economy. Suppose that conditional on Z the default indicators are independent and identically distributed with $P(X_k = 1|Z) = Z$, where $P(Z = 0.01) = 0.9$ and $P(Z = 0.11) = 0.1$. Compute the expected value $E(N)$ where N is defined as in (a).

    (c) Consider the risk factor Z which reflects the state of the economy. Suppose that conditional on Z the default indicators are independent and identically distributed with $P(X_k = 1|Z) = Z^9$, where Z is uniformly distributed on $(0, 1)$. Compute the expected value $E(N)$ where N is defined as in (a).

40. (Exponential tilting for the normal distribution)
    Apply the exponential tilting approach to estimate the tail probability $P(X > c)$ for a standard normally distributed random variable $X \sim N(0, 1)$ and $c >> 0$. Determine the parameter t of the tilted distribution density g_t in this case (cf. lecture).