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Examples of finance instruments affected by credit risk

- bond portfolios
- OTC ("over the counter") transactions
- trades with credit derivatives
- ...

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L is a r.v. and its distribution depends from the c.d.f. of $(X_1, \ldots, X_n, \lambda_1, \ldots, \lambda_n)^T$ ab.

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Then we have $X_i = \begin{cases} 0 & S_i \neq 0 \\ 1 & S_i = 0 \end{cases}$

Models with latent variables (contd.)

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$S = (S_1, S_2, \ldots, S_n)^T$ is modelled by means of latent variables $Y = (Y_1, Y_2, \ldots, Y_n)^T$, e.g. Y_i could be the value of the assets of obligor i

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Let d_{ij}, $i = 1, 2, \ldots, n$, $j = 0, 1, \ldots, m+1$ be threshold values such that $d_{i,0} = -\infty$ und $d_{i,m+1} = \infty$ and $S_i = j \iff Y_i \in (d_{i,j}, d_{i,j+1}]$.

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Let F_i be the distribution function of Y_i. The probability of default for obligor i is $p_i = F_i(d_{i,1})$.

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The probability that the fisrt k obligors default:

$$p_{1,2,\ldots,k} := P(Y_1 \leq d_{1,1}, Y_2 \leq d_{2,1}, \ldots, Y_k \leq d_{k,1})$$

$$= C(F_1(d_{1,1}), F_2(d_{2,1}), \ldots, F_k(d_{k,1}), 1, 1, \ldots, 1) = C(p_1, p_2, \ldots, p_k, 1, \ldots, 1)$$

Thus the totalt defalut probability depends essentially on the copula C of $(Y_1, Y_2, \ldots, Y_n)$.

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Notations:

$V_{A,i}(T)$: value of assets of firm i at time point T

$K_i := K_i(T)$: value of the debt of firm i at time point T

$V_{E,i}(T)$: value of equity of firm i at time point T

Assumption: future asset value is modelled by a geometric Brownian motion

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DD_i is called *distance-to-default*.

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KMVs viewpoint: the equity holders have the right, but not the obligation, to pay off the holders of the other liabilities and take over the remaining assets of the firm.

This can be seen as a call option on the firms assets with a strike price equal to the book value of the firms liabilities.

Thus $V_{E,i}(T) = \max\{V_{A,i}(T) - K_i, 0\}$.

The Black-Scholes formula implies (option price theory):
$V_{E,i}(t) = C(V_{A,i}(t), r, \sigma_{A,i}) = V_{A,i}(t)\phi(e_1) - K_i e^{-r(T-t)}\phi(e_2)$, where
$e_1 = \frac{\ln(V_{A,i}(t) - \ln K_i + (r + \sigma_{A,i}^2/2)(T-t)}{\sigma_{A,i}(T-t)}$, $e_2 = e_1 - \sigma_{A,i}(T-t)$,
ϕ is the the standard normal distribution function and r is the risk free interest rate.

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$$DD_i = \frac{\ln V_{A,i}(t) - \ln K_i + (\mu_{A,i} - \frac{\sigma_{A,i}^2}{2})(T-t)}{\sigma_{A,i}\sqrt{T-t}}.$$

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Then $P(V_{A,i}(T) < K_i) = P(Y_i < -DD_i)$ and in the general setup of the latent variable model with $m = 1$ we have $d_{i1} = -DD_i$.

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Summary of the univariate KMV model to compute the default probability of a company:

- Estimate the asset value $V_{A,i}$ and the volatilty $\sigma_{A,i}$ by using observations of the market value and the volatility of equity $V_{E,i}$, $\sigma_{E,i}$, the book of liabilities K_i, and by solving the system of equations above.
- Compute the distance-to-default DD_i by means of the corresponding formula.
- Estimate the default probability p_i in terms of the empirical distribution which relates the distance to default with the expected default frequency.

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where

$\mu_{A,i}$ is the drift, $\sigma_{A,i}^2 = \sum_{j=1}^{n} \sigma_{A,i,j}^2$ is the volatility, and $\sigma_{A,i,j}$ quantifies the impact of the jth Brownian motion on the asset value of firm i, $i, j \in \{1, 2, \ldots, n\}$.

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Set $Y_i := \frac{\sum_{j=1}^{m} \sigma_{A,i,j}(W_j(T) - W_j(t))}{\sigma_{A,i}\sqrt{T-t}}$. Then $Y = (Y_1, Y_2, \ldots, Y_n) \sim N_n(0, \Sigma)$,

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We get $V_{A,i}(T) < K_i \iff Y_i < -DD_i$ for all $i \in \{1, 2, \dots, n\}$ with

$$DD_i = \frac{\ln V_{A,i}(t) - \ln K_i + \left(\frac{-\sigma_{A,i}^2}{2} + \mu_{A,i}\right)(T-t)}{\sigma_{A,i}\sqrt{T-t}}.$$

The multivariate KMV model (contd.)

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The probability that the k first firms default:

$$P(X_1 = 1, X_2 = 1, \dots, X_k = 1) = P(Y_1 < -DD_1, \dots, Y_k < -DD_k)$$
$$= C_{\Sigma}^{Ga}(\phi(-DD_1), \dots, \phi(-DD_k), 1, \dots, 1),$$

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Joint default frequency:

$$JDF_{1,2,\ldots,k} = C_\Sigma^{Ga}(EDF_1, EDF_2, \ldots, EDF_k, 1, \ldots, 1),$$
where EDF_i is the default frequency for firm i, $i = 1, 2, \ldots, k$.

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Factor model for the latent variables in which the asset value of a company depends on certain common factors (macro-economical, global, regional, sector-based or country-based factors) and a company specific factor.

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$Y = (Y_1, Y_2, \ldots, Y_n)^T = AZ + BU$ where
$Z = (Z_1, \ldots, Z_k)^T \sim N_k(0, \Lambda)$ are the k common factors,
$U = (U_1, \ldots, U_n)^T \sim N_n(0, I)$ are the company specific factors such that Z and U are independent, and the constant matrices $A = (a_{ij}) \in \mathbb{R}^{n \times k}$, $B = diag(b_1, \ldots, b_n) \in \mathbb{R}^{n \times n}$ are model parameters.

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Then we have $cov(Y) = A\Lambda A^T + D$ where $D = diag(b_1^2, \ldots, b_n^2) \in \mathbb{R}^{n \times n}$.

Migration based models: Credit Metrics

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Let P be a portfolio consisting of n credits with a fixed holding duration (eg. 1 year). Let S_i be the status variable for debtor i, where the states are $0, 1, \ldots, m$ and $S_i = 0$ corresponds to default.

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Example: Rating system of Standard and Poor's
$m = 7$; $S_i = 0$ means default; $S_i = 1$ or CCC; $S_i = 2$ or B; $S_i = 3$ or BB; $S_i = 4$ or BBB; $S_i = 5$ or A; $S_i = 6$ or AA; $S_i = 7$ or AAA.

Migration based models: Credit Metrics (contd.)

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For each debtor the dynamics of the status variable is modelled by means of a Markov chain with status set $\{0, 1, \ldots, m\}$ and transition matrix P.

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Original state category	state category at the end of the year AAA	AA	A	BBB	BB	B	CCC	default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

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BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
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Recovery rates

In case of default the recovery rate depends on the status category of the defaulting debtor (prior to default). The mean and the standard deviation of the recovery rate are computed based on the historical data observed over time within each state category.

Evaluation of bonds if the status category changes

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The *forward yield curves* for each status category are given as follows (in %):

Status	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.73	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
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The bond pays 6 units at the end of the 4 years 1, 2, 3, 4 and 106 unit at the end of year 5.

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Assumption: At the end of the first year the bond is rated as an A bond.
The value at the end of the first year:

$$V = 6 + \frac{6}{1 + 3,73\%} + \frac{6}{(1 + 4,32\%)^2} + \frac{6}{(1 + 4,93\%)^3} + \frac{106}{(1 + 5,32\%)^4} = 108.64$$

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Then use the transition probabilities of the Markov chain (estimated in terms of historical data) to compute the expected value of the bond at the end of the first year.

Value and risk of a bond portfolio in Credit Metrics

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Let d_{Def}, d_{CCC}, ..., $d_{AAA} = +\infty$ be thresholds which define the transitions probabilities of debtor i at the end of the current period as follows:

$P(S_i = 0) = \phi(d_{Def})$, $P(S_i = CCC) = \phi(d_{CCC}) - \phi(d_{Def})$, ...,
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These thresholds can be estimated in terms of historical data.

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These thresholds can be estimated in terms of historical data.

The return of a vector of bonds is modelled as a multivariate normal distribution with correlation matrix R estimated by means of factor models.

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can be then computed by using the Gaussian copula $C^{Ga}_{n,R}$ of $(Y_1, Y_2, \ldots, Y_n)$.

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The risk measures (VaR, CVaR) of the bond portfolio, can be computed by simulation.