Risk and Management: Goals and Perspective

Etymology: Risicare

Risk (Oxford English Dictionary): (Exposure to) the possibility of loss, injury, or other adverse or unwelcome circumstance; a chance or situation involving such a possibility.
Finance: The possibility that an actual return on an investment will be lower than the expected return.

Risk management: is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events or to maximize the realization of opportunities.

Risk management's objective is to assure uncertainty does not deflect the endeavor from the business goals.

Risk and Management: Goals and Perspective

Subject of risk managment:

- Identification of risk sources (determination of exposure)
- Assessment of risk dependencies
- Measurement of risk
- Handling with risk
- Control and supervision of risk
- Monitoring and early detection of risk
- Development of a well structured risk management system

Risk and Management: Goals and Perspective

Main questions addressed by strategic risk managment:

- Which are the strategic risks?
- Which risks should be carried by the company?
- Which instruments should be used to control risk?
- What resources are needed to cover for risk?
- What are the risk adjucted measures of success used as steering mechanisms?

Example 1 *Start capital* $V_0 = 100$

Game: lose or gain 50 € with probability 1/2, respectively.

Capital after the game $V_1 = \begin{cases} 150 & \text{with prob. } 1/2 \\ 50 & \text{with prob } 1/2 \end{cases}$

Let $X := V_1 - V_0$ be the gain and let $L := V_0 - V_1$ be the loss.

The distribution function of the random variable X (L) is called **gain distribution** *(GD)* (**loss distribution** *(LD)*).

$L \geq 0 \Rightarrow$ Risk!

Many people would prefer no gain and no loss with cerrtainty rather than either gain or loss with a probability of 1/2, respectively! Those people are *risk averse*. *Risk aversion!*

The decision to play or not the game depends on the loss distribution which is generally not known.

Even if the loss distribution is known the player would prefer to have a number telling her/him how risky is the game!

Definition 1 *A risk measure* ρ *is a mapping from the random variables (r.v.) to the reals which assigns each r.v.* L *a real number* $\rho(L) \in \mathbb{R}$ *.*

Examples: standard devation, quantile of the loss distribution, . . .

Types of risk

For an organization risk arises through events or activities which could prevent the organization from fulfilling its goals and executing its strategies.

Financial risk:

- Market risk
- Credit risk
- Operational risk
- Liquidity risk, legal (judicial) risk, reputational risk

The goal is to estimate these risks as good as possible, ideally based on the loss distribution (LD).

Regulation and supervision

1974: Establishment of Basel Committee on Banking Supervision (BCBS).

Risk capital depending on GD/LD.
Suggestions and guidelines on the requirements and methods used to *compute the risk capital*. Aims at *internationally accepted standards* for the computation of the risk capital and *statutory dispositions* based on those standards.
Control by the supervision agency.

1988 Basel I: International minimum capital requirements especially with respect to (w.r.t.) credit risk.

1996 Standardised models are formulated for the assessment of market risk with an option to use value at risk (VaR) models in larger banks

2007 Basel II: minimum capital requirements w.r.t. credit risk, market risk and operational risk, procedure of control by supervision agencies, market discipline*.

2010 BASEL III - Improvement and further development of BASEL II w.r.t. applicability, operational riskr und liquidity risk

*see `http://www.bis.org`

Assessment of the loss function

Loss operators

$V(t)$ - Value of portfolio at time t

Time unit Δt

Loss in time interval $[t, t+\Delta t]$: $L_{[t,t+\Delta t]} := -(V(t+\Delta t) - V(t))$

Discretisation of time: $t_n := n\Delta t$, $n = 0, 1, 2, \ldots$

$$L_{n+1} := L_{[t_n, t_{n+1}]} = -(V_{n+1} - V_n), \text{ where } V_n := V(n\Delta t)$$

Example 2 *An asset portfolio*

The portfolio consists of α_i units of asset A_i, $i = 1, 2, \ldots, d$.

$S_{n,i}$ price of asset i at time n.

$V_n = \sum_{i=1}^{d} \alpha_i S_{n,i}$

Let $Z_{n,i} := \ln S_{n,i}$, $X_{n+1,i} := \ln S_{n+1,i} - \ln S_{n,i}$

Let $w_{n,i} := \alpha_i S_{n,i}/V_n$, $i = 1, 2, \ldots, d$, be the relative portfolio weights.

Loss operator of an asset portfolio (cont.)

The following holds:

$$L_{n+1} := -\sum_{i=1}^{d} \alpha_i S_{n,i}\Big(\exp\{X_{n+1,i}\}-1\Big) = -V_n \sum_{i=1}^{d} w_{n,i}\Big(\exp\{X_{n+1,i}\}-1\Big) =: l_n(X_{n+1})$$

Linearization $e^x = 1 + x + o(x^2) \sim 1 + x$

$$L^{\triangle}_{n+1} = -V_n \sum_{i=1}^{d} w_{n,i} X_{n+1,i} =: l^{\triangle}_n(X_{n+1}),$$

where $L^{\triangle}$ is the loss function and $l^{\triangle}$ is the loss operator.

The general case

Let $V_n = f(t_n, Z_n)$ and $Z_n = (Z_{n,1}, \ldots, Z_{n,d})$, where Z_n is a vector of risk factors

Risk factor changes: $X_{n+1} := Z_{n+1} - Z_n$

$L_{n+1} = -\Big(f(t_{n+1}, Z_n + X_{n+1}) - f(t_n, Z_n)\Big) =: l_n(X_{n+1})$, where

$l_n(x) := -\Big(f(t_{n+1}, Z_n + x) - f(t_n, Z_n)\Big)$ is the loss operator.

The linearized loss:

$L^{\Delta}_{n+1} = -\Big(f_t(t_n, Z_n)\Delta t + \sum_{i=1}^{d} f_{z_i}(t_n, Z_n) X_{n+1,i}\Big)$,

where f_t and f_{z_i} are the partial derivatives of f.

The linearized loss operator:

$l^{\Delta}_n(x) := -\Big(f_t(t_n, Z_n)\Delta t + \sum_{i=1}^{d} f_{z_i}(t_n, Z_n) x_i\Big)$

Financial derivatives are financial products or contracts, which are based on a fundamental basic product (zB. asset, asset index, interest rate, commodity) and are derived from it

Definition 2 *A* European call option (ECO) *on a certain asset S grants its holder the right but not the obligation to buy asset S at a specified day T (execution day) and at a specified price K (strike price). The option is bought by the owner at a certain price at day* 0*.*

Value of ECO at time t: $C(t) = \max\{S(t) - K, 0\}$,
where $S(t)$ is the market price of asset S at time t.

Definition 3 *Eine zero-coupon bond (ZCB) with maturity T is a contract, which gives the holder of the contract 1€ at time T. The price of the contract at time t is denoted by $B(t,T)$. By definition $B(T,T) = 1$.*

Definition 4 *A currency forward or an FX forward (FXF) is a contrcat between two parties to buy/sell an amount $\bar{V}$ of foreign currency at a future time T for a specified exchange rate $\bar{e}$. The party who is going to buy the foreign currency is said to hold a long position and the party who will sell holds a short position.*

Example 3 *A bond portfolio*

Let $B(t,T)$ be the price of the zero-coupon bond with maturity T at time $t < T$.

The *continuously compounded yield*, $y(t,T) := -\frac{1}{T-t} \ln B(t,T)$, would represent the continuous interest rate which was dealt with at time t as being constant for the whole interval $[t,T]$.

There are different yields for different maturities.

The *yield curve* for fixed t is a function $T \mapsto y(t,T)$.

Consider a portfolio consisting of α_i units of ZCB i with maturity T_i and price $B(t,T_i)$, $i = 1,2,\ldots,d$.

Portfolio value:

$$V_n = \sum_{i=1}^{d} \alpha_i B(t_n, T_i) = \sum_{i=1}^{d} \alpha_i exp\{-(T_i - t_n) Z_{n,i}\} = f(t_n, Z_n)$$

where $Z_{n,i} := y(t_n, T_i)$ are the risk factors.
Let $X_{n+1,i} := Z_{n+1,i} - Z_{n,i}$ be the risk factor changes.

$$l_{[n]}(x) = -\sum_{i=1}^{d} \alpha_i B(t_n, T_i) \left(exp\{Z_{n,i}\Delta t - (T_i - t_{n+1})x_i\} - 1\right)$$

$$L^{\Delta}_{n+1} = -\sum_{i=1}^{d} \alpha_i B(t_n, T_i) \left(Z_{n,i}\Delta t - (T_i - t_{n+1})X_{n+1,i}\right)$$

Example 4 *A currency forward portfolio*

The party who buys the foreign currency holds a *long position*. The party who sells holds a *short position*.

Long position over $(\bar{V})$ units of a FX forward with maturity T
$\Longleftrightarrow$
Long position over $\bar{V}$ units of a foreign zero-coupon bond (ZCB) with maturity T
and
a short position over $\bar{e}\bar{V}$ units of a domestic zero-coupon bond with maturity T.

Assumptions:

Euro investor holds a long position of a USD/EUR forward over $\bar{V}$ USD.

Let $B^f(t,T)$ $(B^d(t,T))$ be the price of a USD based (EUR-based) ZCB.

Let $e(t)$ be the spot exchange rate for USD/EUR.

Value of the long position of the FX forward at time T : $V_T = \bar{V}(e(T) - \bar{e})$.

The short position of the domestic ZCB can be handled as in Example (3). The long losition in the foreign ZCB:

Risk factors: $Z_n = (\ln e(t_n), y^f(t_n,T))^T$

Value of the long position (in Euro): $V_n = \bar{V} \exp\{Z_{n,1} - (T - t_n)Z_{n,2}\}$

The linearized loss: $L^{\Delta}_{n+1} = -V_n(Z_{n,2}\Delta t + X_{n+1,1} - (T - t_{n+1})X_{n+1,2})$

where $X_{n+1,1} := \ln e(t_{n+1}) - \ln e(t_n)$ und $X_{n+1,2} := y^f(t_{n+1},T) - y^f(t_n,T)$