Table 8.1 *Total returns for stocks, bonds, and money market*

Year	Stocks	Bonds	MM	Year	Stocks	Bonds	MM
1960	20.2553	262.935	100.00	1982	115.308	777.332	440.68
1961	25.6860	268.730	102.33	1983	141.316	787.357	482.42
1962	23.4297	284.090	105.33	1984	150.181	907.712	522.84
1963	28.7463	289.162	108.89	1985	197.829	1200.63	566.08
1964	33.4484	299.894	113.08	1986	234.755	1469.45	605.20
1965	37.5813	302.695	117.97	1987	247.080	1424.91	646.17
1966	33.7839	318.197	124.34	1988	288.116	1522.40	702.77
1967	41.8725	309.103	129.94	1989	379.409	1804.63	762.16
1968	46.4795	316.051	137.77	1990	367.636	1944.25	817.87
1969	42.5448	298.249	150.12	1991	479.633	2320.64	854.10
1970	44.2212	354.671	157.48	1992	516.178	2490.97	879.04
1971	50.5451	394.532	164.00	1993	568.202	2816.40	905.06
1972	60.1461	403.942	172.74	1994	575.705	2610.12	954.39
1973	51.3114	417.252	189.93	1995	792.042	3287.27	1007.84
1974	37.7306	433.927	206.13	1996	973.897	3291.58	1061.15
1975	51.7772	457.885	216.85	1997	1298.82	3687.33	1119.51
1976	64.1659	529.141	226.93	1998	1670.01	4220.24	1171.91
1977	59.5739	531.144	241.82	1999	2021.40	3903.32	1234.02
1978	63.4884	524.435	266.07	2000	1837.36	4575.33	1313.00
1979	75.3032	531.040	302.74	2001	1618.98	4827.26	1336.89
1980	99.7795	517.860	359.96	2002	1261.18	5558.40	1353.47
1981	94.8671	538.769	404.48	2003	1622.94	5588.19	1366.73

Table 8.2 *Rates of return for stocks, bonds and money market*

Year	Stocks	Bonds	MM	Year	Stocks	Bonds	MM
1961	26.81	2.20	2.33	1983	22.56	1.29	9.47
1962	−8.78	5.72	2.93	1984	6.27	15.29	8.38
1963	22.69	1.79	3.38	1985	31.17	32.27	8.27
1964	16.36	3.71	3.85	1986	18.67	22.39	6.91
1965	12.36	0.93	4.32	1987	5.25	−3.03	6.77
1966	−10.10	5.12	5.40	1988	16.61	6.84	8.76
1967	23.94	−2.86	4.51	1989	31.69	18.54	8.45
1968	11.00	2.25	6.02	1990	−3.10	7.74	7.31
1969	−8.47	−5.63	8.97	1991	30.46	19.36	4.43
1970	3.94	18.92	4.90	1992	7.62	7.34	2.92
1971	14.30	11.24	4.14	1993	10.08	13.06	2.96
1972	18.99	2.39	5.33	1994	1.32	−7.32	5.45
1973	−14.69	3.29	9.95	1995	37.58	25.94	5.60
1974	−26.47	4.00	8.53	1996	22.96	0.13	5.29
1975	37.23	5.52	5.20	1997	33.36	12.02	5.50
1976	23.93	15.56	4.65	1998	28.58	14.45	4.68
1977	−7.16	0.38	6.56	1999	21.04	−7.51	5.30
1978	6.57	−1.26	10.03	2000	−9.10	17.22	6.40
1979	18.61	−1.26	13.78	2001	−11.89	5.51	1.82
1980	32.50	−2.48	18.90	2002	−22.10	15.15	1.24
1981	−4.92	4.04	12.37	2003	28.68	0.54	0.98
1982	21.55	44.28	8.95				

8.1 Mean-variance optimization

Covariance	Stocks	Bonds	MM
Stocks	0.02778	0.00387	0.00021
Bonds	0.00387	0.01112	−0.00020
MM	0.00021	−0.00020	0.00115

It is interesting to compute the volatility of the rate of return on each asset $\sigma_i = \sqrt{\text{cov}(R_i, R_i)}$:

	Stocks	Bonds	MM
Volatility	16.67%	10.55%	3.40%

and the correlation matrix $\rho_{ij} = \frac{\text{COV}(R_i, R_j)}{\sigma_i \sigma_j}$:

Correlation	Stocks	Bonds	MM
Stocks	1	0.2199	0.0366
Bonds	0.2199	1	−0.0545
MM	0.0366	−0.0545	1

Setting up the QP for portfolio optimization:

$$\min 0.02778x_S^2 + 2 \cdot 0.00387x_S x_B + 2 \cdot 0.00021x_S x_M$$
$$+0.01112x_B^2 - 2 \cdot 0.00020x_B x_M + 0.00115x_M^2$$
$$0.1073x_S + 0.0737x_B + 0.0627x_M \geq R \qquad (8.6)$$
$$x_S + x_B + x_M = 1$$
$$x_S \geq 0, x_B \geq 0, x_M \geq 0,$$

Table 8.3 *Efficient portfolios*

Rate of return R	Variance	Stocks	Bonds	MM
0.065	0.0010	0.03	0.10	0.87
0.070	0.0014	0.13	0.12	0.75
0.075	0.0026	0.24	0.14	0.62
0.080	0.0044	0.35	0.16	0.49
0.085	0.0070	0.45	0.18	0.37
0.090	0.0102	0.56	0.20	0.24
0.095	0.0142	0.67	0.22	0.11
0.100	0.0189	0.78	0.22	0
0.105	0.0246	0.93	0.07	0
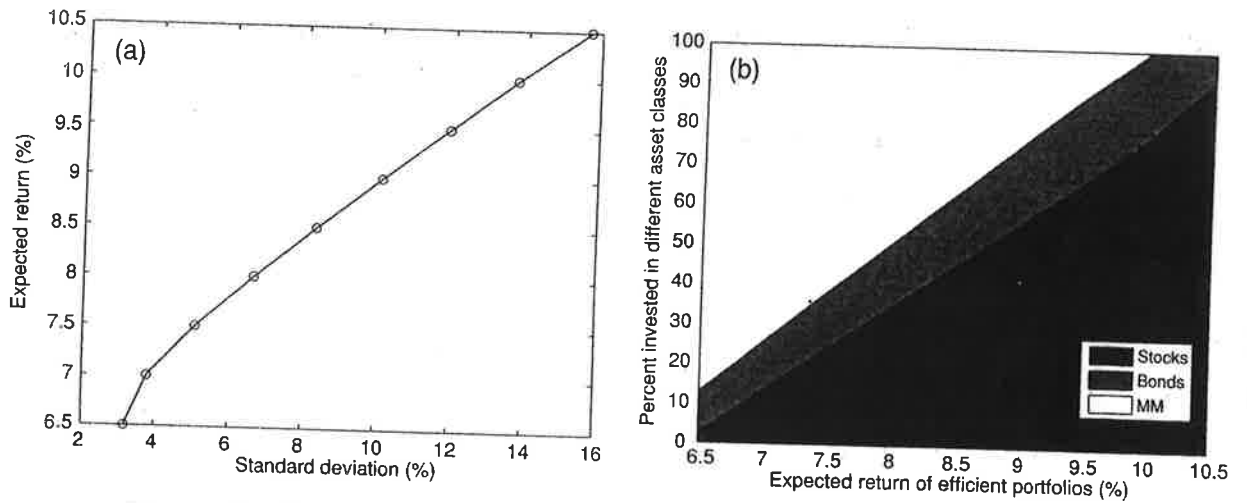
Figure 8.1 Efficient frontier and the composition of efficient portfolios

Table 8.6 *Konno–Yamazaki efficient portfolios*

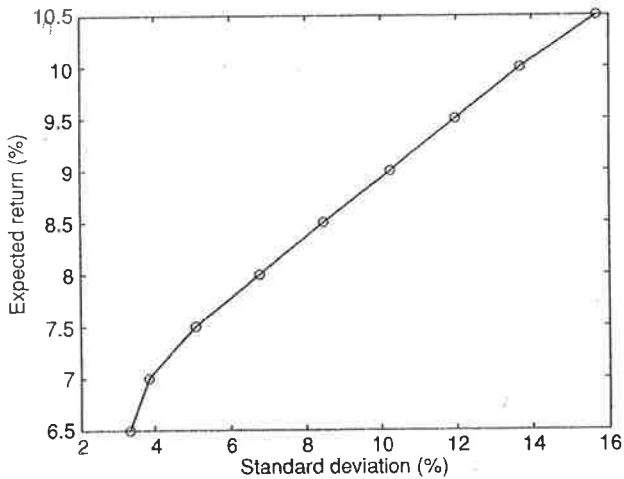| Rate of return R	Variance	Stocks	Bonds	MM
0.065	0.0011	0.05	0.01	0.94
0.070	0.0015	0.15	0.04	0.81
0.075	0.0026	0.25	0.11	0.64
0.080	0.0046	0.32	0.28	0.40
0.085	0.0072	0.42	0.32	0.26
0.090	0.0106	0.52	0.37	0.11
0.095	0.0144	0.63	0.37	0
0.100	0.0189	0.78	0.22	0
0.105	0.0246	0.93	0.07	0
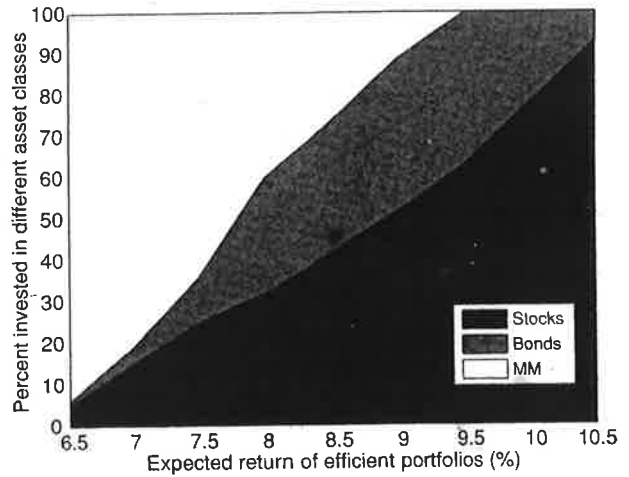
Figure 8.3 Efficient frontier and the composition of efficient portfolios using the Konno–Yamazaki approach

Table 8.5 *Black–Litterman efficient portfolios*

| Rate of return R	Variance	Stocks	Bonds	MM
0.040	0.0012	0.08	0.17	0.75
0.045	0.0015	0.11	0.21	0.68
0.050	0.0020	0.15	0.24	0.61
0.055	0.0025	0.18	0.28	0.54
0.060	0.0032	0.22	0.31	0.47
0.065	0.0039	0.25	0.35	0.40
0.070	0.0048	0.28	0.39	0.33
0.075	0.0059	0.32	0.42	0.26
0.080	0.0070	0.35	0.46	0.19
0.085	0.0083	0.38	0.49	0.13
0.090	0.0096	0.42	0.53	0.05
0.095	0.0111	0.47	0.53	0
0.100	0.0133	0.58	0.42	0
0.105	0.0163	0.70	0.30	0
0.110	0.0202	0.82	0.18	0
0.115	0.0249	0.94	0.06	0
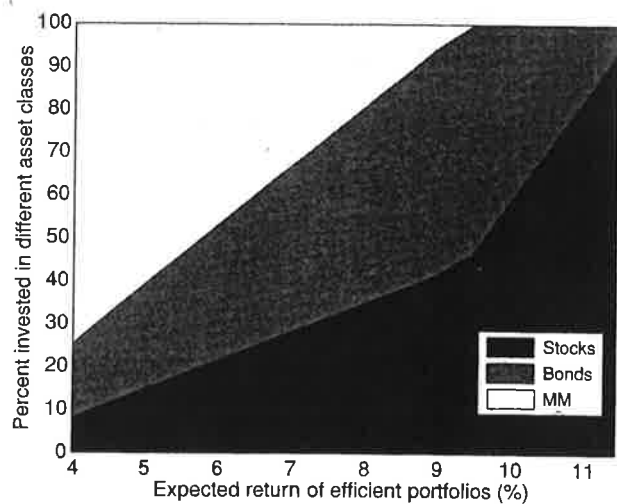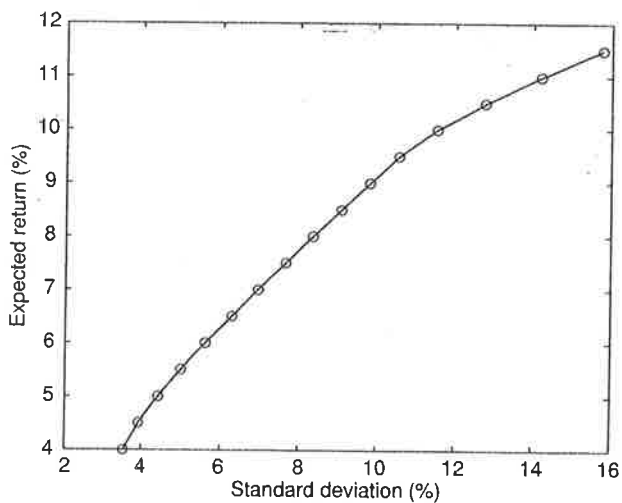
Figure 8.2 Efficient frontier and the composition of efficient portfolios using the Black–Litterman approach