

**Operations Research**  
**winter term 2017/2018**

**4th work sheet (inventory models)**

24. Consider an inventory system with three products in inventory. The fixed ordering costs  $K_i$ , the variable ordering costs  $c_i$  per piece, the inventory costs  $h_i$  per piece and month, and the demands rates  $\mu_i$  for product  $i$ ,  $i \in \{1, 2, 3\}$ , are given in the following table:

	Product 1	Product 2	Product 3
$K$ (in Euro)	40	10	45
$c$ (in Euro per piece)	0.5	1	0.25
$h$ (in Euro per piece and month)	0.1	0.2	0.05
$\mu$ (in pieces per month)	200	100	50

- (a) Assume that the cycle length should be the same for all products, such that all products can be delivered at the same time. Determine the optimal lot sizes, the optimal cycle length and the optimal (i.e. minimal) overall ordering and inventory costs per product.
- (b) Assume that the value of the products in inventory should not exceed a certain amount  $\gamma$ . Determine the optimal lot sizes, the optimal cycle length and the optimal (i.e. minimal) overall ordering and inventory costs per product for  $\gamma = 200$  Euro,  $\gamma = 150$  Euro and  $\gamma = 30$  Euro, respectively.
25. Assume that the demand on a particular spare part of an old aeroplane type is exponentially distributed with expected value equal to 50. The production of the old aeroplane type will be shut down in one year of time and thus the whole production of spare parts which will be eventually needed should ideally happen during the current year. The variable production costs amount to 1000 Euro per piece if the production happens in the current year. In the case that the production happens at a later point in time, the variable production costs amount to 10000 Euro per piece. There are no fixed production costs. The inventory costs for the spare parts remaining in inventory at the end of the year amount to 300 per piece.
- (a) Determine the optimal number of the spare parts to be produced such that the expected overall costs are minimized (with the settings of a standard one period model including a starting inventory level of 0 pieces).
- (b) Determine the optimal number of the spare parts to be produced such that the expected overall costs are minimized if the initial inventory level is 23 pieces.
- (c) Assume that the shortage costs cannot be estimated yet at the moment. How many spare parts should be produced such that a shortage will be realized with probability of at most 0.1 at the end of the one year period.
- (d) Consider the original standard model including shortage costs. Assume that the lot size computed in point (c) is optimal for this model and compute the shortage costs per piece and year such that this assumption holds.
26. The stochastic one-period inventory model is extended as follows. There are two products in inventory. They can serve as a substitute for each-other, i.e. Product 1 (Product 2) can be used to fulfill the demand on Product 2 (Product 1), in the case that the demand cannot be fulfilled by the inventory of the specific product. Assume for simplicity that the inventory level of each product equals 0 at the beginning of the planning horizon. Moreover assume that both the fixed ordering costs and the costs of missing demand equal 0. Let  $c_i$  be the variable ordering cost per unit of product  $i$  and let  $s_i$  be the profit realized by fulfilling the demand of one unit of product  $i$ ,  $i = 1, 2$ . Hence the fulfillment of a demand for  $z$  units of product  $i$  results in a net profit of  $z(s_i - c_i)$ ,  $i = 1, 2$ , for the

inventory holder. Let  $X_i$  be independent random variables representing the demand on product  $i$  and let  $f_i$  be the probability density function of  $X_i$ , for  $i = 1, 2$  respectively. Further let  $a_i \in (0, 100)$  be the percentage of customers which would eventually accept a substitute for their demand. Denote by  $G(u_1, u_2)$  the (random) profit of the inventory holder if he orders  $u_i$  units of product  $i$ ,  $i = 1, 2$ . The goal is to determine the amounts  $u_i$  of products to be ordered,  $i = 1, 2$ , such that the expected profit  $E(G(u_1, u_2))$  is maximized. Derive a formula for  $E(G(u_1, u_2))$ .

27. Consider a three-period stochastic stationary inventory model as the one dealt with in the lecture (version  $A$  and  $B$ ). Let the random demand  $R$  be uniformly distributed over the interval  $[0, 10]$ . The other input parameters of the problem are as follows:  $c = 1$  Euro per product unit,  $K = 5$  Euro,  $h = 3$  Euro per product unit and period,  $p = 4$  Euro per product unit and period, and  $\alpha = 0.9$ . Determine an optimal ordering policy  $z_j^*(x)$ ,  $x \in \mathbb{R}$ ,  $j = 1, 2, 3$ , and the corresponding costs, for each of the models  $A$  and  $B$  (cf. lecture). Determine also the lower and upper bounds for the ordering point and the ordering level in every period.
28. Consider a stochastic stationary inventory model with an infinite number of periods as described in the lecture. Let the random demand  $R$  be uniformly distributed over the interval  $[0, 10]$ . The other input parameters of the problem are as follows:  $c = 1$  Euro per product unit,  $K = 0$  Euro,  $h = 3$  Euro per product unit and period,  $p = 4$  Euro per product unit and period, and  $\alpha = 0.9$ . Determine an optimal ordering policy  $z^*(x)$ ,  $x \in \mathbb{R}$  and the corresponding costs.