Operations Research
winter term 2019/2020

4th work sheet (dynamic programming)

17. Consider an instance of the Knapsack Problem (KP) with $n = 6$ objects, profit vector $p = (3, 2, 1, 2, 3, 1)$, weight vector $w = (3, 4, 2, 5, 4, 3)$ and weight limit $W = 14$.

    (a) Apply the greedy algorithm to solve the corresponding instance of the Fractional Knapsack Problem (FKP).

    (b) Apply the dynamic programming algorithm discussed in the lecture to solve the above instance of the Knapsack Problem.

    (c) A feasible solution (consisting of a subset S of the set of objects $\{1, 2, \ldots, n\}$) of the KP is called a *c-approximation* of an optimal solution $S^* \subset \{1, 2, \ldots, n\}$, for $c \in \mathbb{R}$, $c > 1$, iff $\sum_{i \in S^*} p_i \le c \sum_{i \in S} p_i$ holds.
    Observe that the optimal objective function value of the KFP is an upper bound on the optimal objective function value of the KP. Based on this observation show how to construct a 2-approximation of the optimal solution of the KP based on the greedy optimal solution of the FKP and the critical object. Apply this idea to the given instance of KP.

18. Consider the m-Dimensional Knapsack Problem (m-KP) defined as follows. The instance consists of a natural number n and nonnegative integers p_i, w_{ij}, W_j, for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. The task is to find a subset $S \subseteq \{1, 2 \ldots, n\}$ such that $\sum_{i=1}^{n} w_{ij} \le W_j$ holds, for all $j \in \{1, 2, \ldots, m\}$ and $\sum_{i=1}^{n} c_i$ is maximized. Give a dynamic programming algorithm for the m-Dimensional Knapsack Problem

19. Six crew members of a sales division will be assigned to three regions. Every region should become at least one crew meber to supervise it and every crew member should supervise just one region. Table 19 shows an estimation (in a suitable unit) for the increase of the business volume in each region depending on the number of crew members assigned to it. Determine how many crew members should be assigned to each reagion such that the overall increase of the business volume is maximized. Solve this problem by means of dynamic programming.

Number of	Region		
crew members	1	2	3
1	4	3	5
2	6	6	7
3	9	8	10
4	11	10	12

Table 1: Data for problem 19