Operations Research
winter term 2019/2020

2nd work sheet (integer programming)

8. Consider a linear integer program with a minimization objective function solved by a branch and bound algorithm. Assume that at the root node of the branch and bound tree the following optimal simplex tableau was obtained.

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$-z$	$-25/3$	0	$4/3$	$16/9$	$9/2$	0	0	0	$7/6$
x_5	1	0	1	$-1/2$	$-3/2$	1	0	0	$3/2$
x_1	$11/3$	1	$-2/3$	$-1/3$	0	0	0	0	$2/3$
x_6	$2/3$	0	$1/3$	$1/6$	$-1/2$	0	1	0	$7/6$
x_7	1	0	-3	$1/2$	$9/2$	0	0	1	$-15/2$

Choose x_1 as the branching variable and process the first level of the branching tree, i.e. the two child-nodes of the root. For each child-node solve the corresponding linear program by means of the dual simplex algorithm starting from the optimal tableau of the parent node. After having processed the two child-nodes stop processing the branching tree and report the range within which the value of the optimal solution lies, i.e. report global lower and upper bounds for the value of the optimal solution.

9. Solve the following integer program using implicit enumeration

$$\begin{aligned}
\text{maximize} \quad & 2x_1 - x_2 - x_3 + 10\,, \\
\text{subject to} \quad & \\
& 2x_1 + 3x_2 - x_3 \le 9\,, \\
& x_1 + 2x_2 + 3x_3 \ge 4\,, \\
& 3x_1 + 3x_2 + 3x_3 = 6\,, \\
& x_i \in \{0,1\}\,, \text{ for } i = 1,2,3.
\end{aligned}$$

In a preprocessing step transform the given binary linear program to an equivalent binary linear program with the following properties:

(i) the goal is a maximization objective function,

(ii) all coefficients in the objective function are non-positive, except for the constant term for which no sign restrcitions hold,

(iii) each constraint is of the form $a^t x \le b$ for some vector $a \in \mathbb{R}^3$ and some constant b.

Then solve the transformed problem by explicit enumeration and transform its optimal solution to an optimal solution of the original problem. Collect the subproblems in a tree, specify the local and global lower and upper bounds in every node of the tree and explain what kind of fathoming is performed (if there is performed any fathoming at all).

Hint: In order to fulfill to (ii) a variable transformation of the form $x' = 1 - x$ can be performed.

10. The following tableau specifies an optimal solution to the linear program LP associated with the integer program IP presented below.

Basic variables	Current values	x_1	x_2	x_3	x_4
x_1	$16/5$	1	0	$2/5$	$-1/5$
x_2	$23/5$	0	1	$1/5$	$2/5$
$-z$	$-133/5$	0	0	$-11/5$	$-2/5$

$$
\begin{aligned}
\text{maximize } z &= 4x_1 + 3x_2\,, \\
\text{subject to} & \\
& 2x_1 + x_2 + x_3 = 11\,, \\
& -x_1 + 2x_2 + x_4 = 6\,, \\
& x_i \geq 0 \text{ and integer, for } i = 1, 2, 3, 4.
\end{aligned}
$$

(i) Define cuts from each of the rows in the optimal linear programming tableau, including the objective function.

(ii) Express the cuts in terms of the variables x_1 and x_2.

(iii) Append the cut derived from the objective function to the linear program LP and resolve the extended linear program LP' by using the dual simplex method. To this end you can extend the optimal basis of LP given in the tableau above by the slack variable of the added cut. This extended basis can serve as a starting basis for the dual simplex method applied to problem LP'.

(iv) Does the solution of LP' solve the original problem IP? If not how would you further proceed to solve IP?

11. The *m-travelling salesman problem* (m-TSP) is a variant of *the travelling salesman problem* (TSP) in which m salespersons stationed at a home base, e.g. city 1, are to make tours to visit other cities 2, 3, ...,n. Each salesperson must be routed to some, but not all, of the cities and return to him (or her) homebase, such that (a) each city, except for the home base, is visited by exactly one salesperson, and (b) no salesperson visits a city more than once. For $i, j \in \{1, 2, \ldots, n\}$, $i \neq j$, let c_{ij} be the cost to travel from city i to city j for each salesperson.

(i) Formulate an integer program to determine the minimum cost routing plan. Be sure that your formulation does not permit subtour solutions for any salesperson.

(ii) The m-TSP can be reformutaed as a TSP as follows. Replace the homebase (city 1) by m fictious cities denoted by $1'$, $2'$, ..., m'. Link each of those fictious cities to each other at a large cost $M > \sum_{i=1}^{n} \sum_{j=1}^{n} |c_{ij}|$. Further connect every fictious city i' to every city j at cost $c_{i'j} = c_{1j}$, for any $i' \in \{1', 2', \ldots, m'\}$ and for any $j \in \{1, 2, \ldots, n\}$. Now solve the TSP for the extended instance with $m + n$ cities. How can you transform an optimal solution of the (extended) TSP instance to an optimal solution of the original m-TSP instance? Argue upon you answer carefully.