

### SUCCESSIVE SHORTEST PATH ALGORITHM

*Input:* A digraph  $G$ , capacities  $u : E(G) \rightarrow \mathbb{R}_+$ , numbers  $b : V(G) \rightarrow \mathbb{R}$  with  $\sum_{v \in V(G)} b(v) = 0$ , and conservative weights  $c : E(G) \rightarrow \mathbb{R}$ .

*Output:* A minimum cost  $b$ -flow  $f$ .

- ① Set  $b' := b$  and  $f(e) := 0$  for all  $e \in E(G)$ .
- ② **If  $b' = 0$  then stop, else:**  
Choose a vertex  $s$  with  $b'(s) > 0$ .  
Choose a vertex  $t$  with  $b'(t) < 0$  such that  $t$  is reachable from  $s$  in  $G_f$ .  
**If there is no such  $t$  then stop.** (There exists no  $b$ -flow.)
- ③ Find an  $s$ - $t$ -path  $P$  in  $G_f$  of minimum weight.
- ④ Compute  $\gamma := \min \left\{ \min_{e \in E(P)} u_f(e), b'(s), -b'(t) \right\}$ .  
Set  $b'(s) := b'(s) - \gamma$  and  $b'(t) := b'(t) + \gamma$ . Augment  $f$  along  $P$  by  $\gamma$ .  
**Go to ②.**