

The main idea of the algorithm:

Choose first any $s, t \in V(G)$ and find a min. s - t -cut $\delta(A)$.

Set $B := V(G) \setminus A$. Then contract A to a single vertex and choose $s', t' \in B$. Then find a minimum s' - t' -cut in the contracted graph G' .

Continue this process by always choosing a pair s', t' of vertices which have not been separated by any cut found so far.

At each step we contract A' or B' depending on which part does not contain s' and t' .

GOMORY-HU ALGORITHM

Input: An undirected graph G and a capacity function $u : E(G) \rightarrow \mathbb{R}_+$.

Output: A Gomory-Hu tree T for (G, u) .

- ① Set $V(T) := \{V(G)\}$ and $E(T) := \emptyset$.
- ② Choose some $X \in V(T)$ with $|X| \geq 2$. If no such X exists then go to ⑥.
- ③ Choose $s, t \in X$ with $s \neq t$.
For each connected component C of $T - X$ **do:** Let $S_C := \bigcup_{Y \in V(C)} Y$.
 Let (G', u') arise from (G, u) by contracting S_C to a single vertex v_C for each connected component C of $T - X$.
 (So $V(G') = X \cup \{v_C : C \text{ is a connected component of } T - X\}$.)
- ④ Find a minimum s - t -cut $\delta(A')$ in (G', u') . Let $B' := V(G') \setminus A'$.
 Set $A := \left(\bigcup_{v_C \in A' \setminus X} S_C \right) \cup (A' \cap X)$ and $B := \left(\bigcup_{v_C \in B' \setminus X} S_C \right) \cup (B' \cap X)$.
- ⑤ Set $V(T) := (V(T) \setminus \{X\}) \cup \{A \cap X, B \cap X\}$.
For each edge $e = \{X, Y\} \in E(T)$ incident to the vertex X **do:**
 If $Y \subseteq A$ then set $e' := \{A \cap X, Y\}$ else set $e' := \{B \cap X, Y\}$.
 Set $E(T) := (E(T) \setminus \{e\}) \cup \{e'\}$ and $w(e') := w(e)$.
 Set $E(T) := E(T) \cup \{\{A \cap X, B \cap X\}\}$.
 Set $w(\{A \cap X, B \cap X\}) := u'(\delta_{G'}(A'))$.
Go to ②.
- ⑥ Replace all $\{x\} \in V(T)$ by x and all $\{\{x\}, \{y\}\} \in E(T)$ by $\{x, y\}$. **Stop.**

At the end each pair of vertices is separated after a total of $n-1$ cut (i.e. iterations) because in every iteration the number of vertices decreases by at least 1.

Crucial observation: A minimum s' - t' -cut in the contracted graph G' is also a minimum s - t -cut in G .