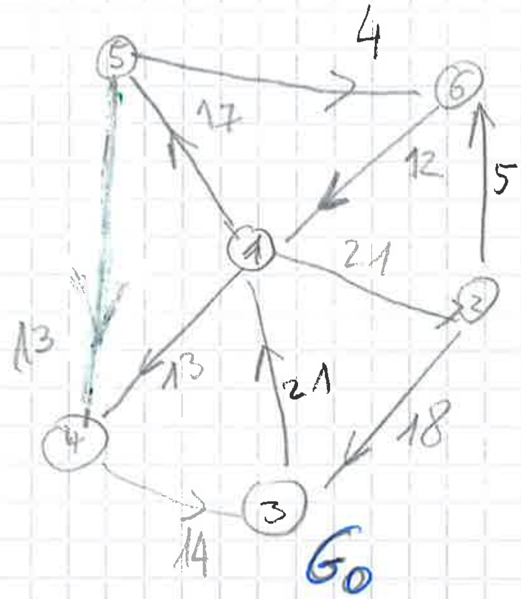
2021 B2A 10

Example for the Edmonds branching algorithm

1



G_0

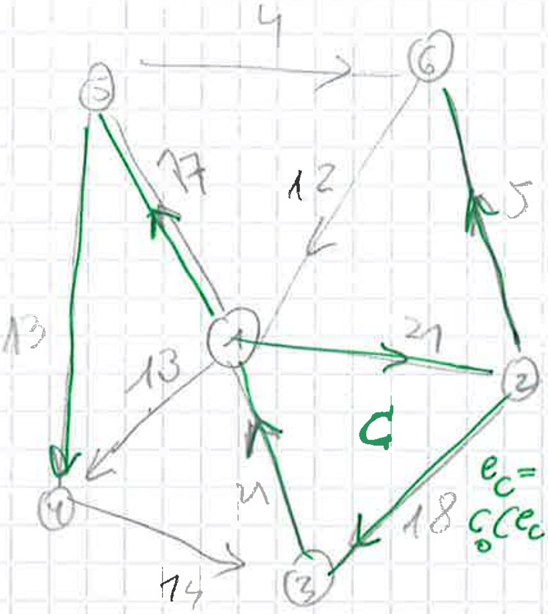
Step 1

$G = G_0 \quad i = 0 \quad c_0 = c$

Step 2 : Construct B_0

Step 3 . B_0 contains a cycle $(1,2,3) = C$

$B := B_0$



B_0 Contract the circuits of G_0 and obtain G_1 with its weights c_1 and the representatives $\phi_1(\cdot)$

$e_C = 2,3$

$c_0(e_C) = 18$

$\phi_1(5,6) = (5,6)$

$\phi_1(5,4) = (5,4)$

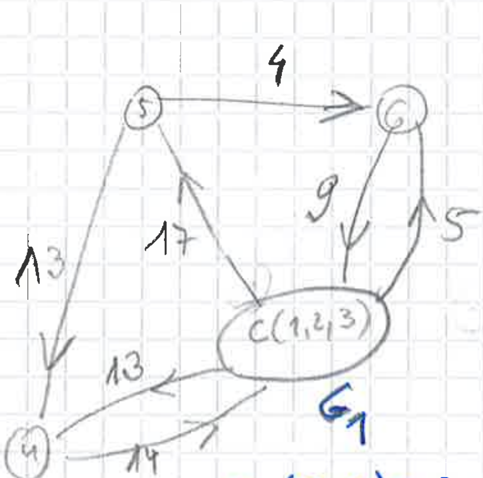
$\phi_1(6, C(1,2,3)) = (6,1)$

$\phi_1(C(1,2,3), 6) = (2,6)$

$\phi_1(C(1,2,3), 5) = (1,5)$

$\phi_1(C(1,2,3), 4) = (1,4)$

$\phi_1(4, C(1,2,3)) = (4,3)$



G_1

$c_1(5,6) = c_0(5,6) \qquad c_1(5,4) = c_0(5,4)$

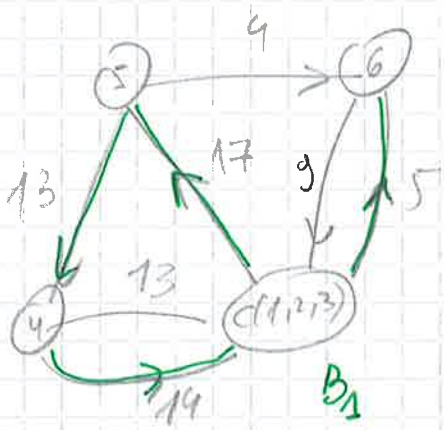
$c_1(C(1,2,3), 5) = c_0(1,5) \qquad c_1(C(1,2,3), 6) = c_0(2,6)$

$c_1(C(1,2,3), 4) = c_0(1,4) = 13 \qquad c_1(4, C(1,2,3)) = c_0(4,3) - c_0(2,3) + c_0(e_C) = 14$

$c_1(6, C(1,2,3)) = c_0(6,1) - c_0(3,1) + c_0(e_C) = 9$

Step 3: Second iteration: Set $i = 1$

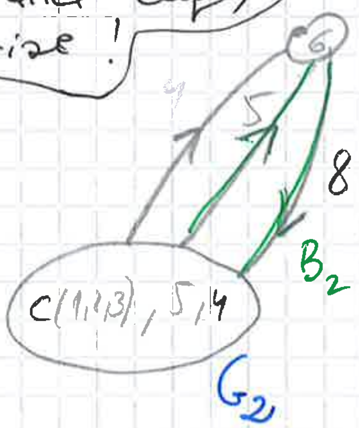
Construct B_1



B_1 has a cycle $C(C(1,2,3), 5, 4)$

Contract it to obtain G_2 and its weights c_2 and the representatives $\phi_2(\cdot)$

Notice parallel edges arise!



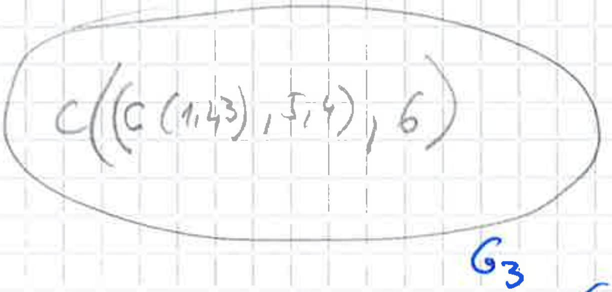
Step 3: third iteration

set $i = 2$

Construct B_2

B_2 has one cycle $C((C(1,2,3), 5, 4), 6)$

Contract it to obtain G_3 and the weights c_3 and the representatives $\phi_3(\ )$.



Step 3: fourth iteration

Set $i = 3$

Construct B_3

Since $E(G_3) = \phi$ also $B_3 = \phi$.

So B_3 is cycle-free. Goto Step 5

Set $B_i = B_3$

2021 B&A 12

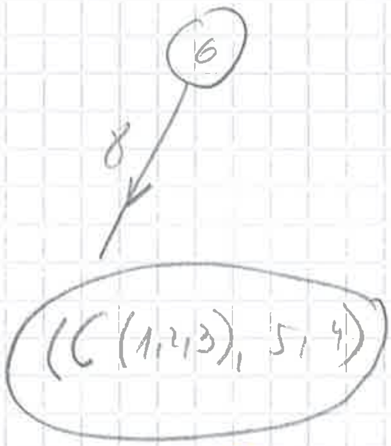
$i = 3 > 0$

let $B' = (V(G_2), \{\phi_3(e) : e \in \underbrace{E(B)}_{=\emptyset}\}) = (V(G_2), \emptyset)$

$\forall$ cycle C in B_2 do $[C = ((c(1,2,3), 5, 4), 6)]$

$E(B') = \underbrace{E(B')}_{=\emptyset} \cup (E(C) \setminus \{e_C\}) = (6, (c(1,2,3), 5, 4))$

$B := B'$ $\qquad i = 3 - 1 = 2$



B

Step 5, second iteration

$B' = (V(G_1), \{\phi_2(e) : e \in E(B)\})$

$V(G_1) = \{c(1,2,3), 4, 5, 6\}$ $\qquad \{\phi_2(e) : e \in E(B)\} = \{(6, c(1,2,3))\}$

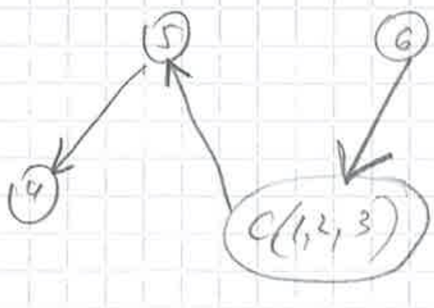
$\forall$ cycle C of B_1 (which is $(c(1,2,3), 5, 4)$)

if there is an edge entering the cycle then

$E(B') = E(B') \cup (E(C) \setminus \{(4, c(1,2,3))\}) =$

$= \{(6, c(1,2,3)), (5,4), (c(1,2,3), 5)\}$

$B = B'$ $\qquad i = 2 - 1 = 1$



B

Step 5, third iteration

$i = 1 > 0$

$B' = \big(V(G_0) , \{ \phi_1(e) : e \in E(B) \} \big)$

$\{1,2,3,4,5,6\}$, $\{ (5,4), (1,5), (6,1) \}$

∀ circuit C in B_0 (which is $C = C(1,2,3)$) do

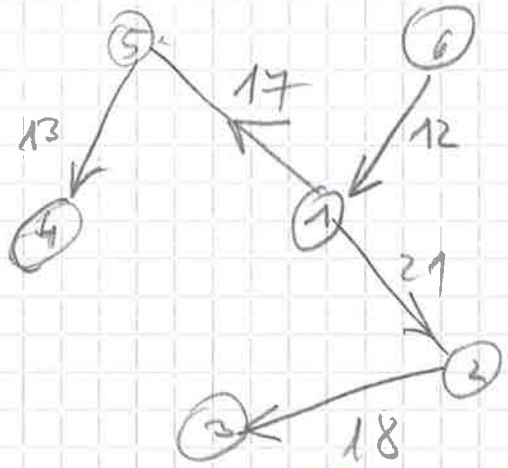
, if there is an edge entering C (which is $(6,1)$)

set $E(B') := E(B') \cup \big(E(C) \setminus \{(2,4)\} \big)$

$= \{ (5,4), (1,5), (6,1), (1,2), (2,3) \}$

Set $B = B'$ $\qquad i = i - 1 = 0$

STOP



B

B is the maximum Branching with weight $13 + 17 + 12 + 18 + 31$
$= 91$