

EDMONDS' BRANCHING ALGORITHM

Input: A digraph G , weights $c : E(G) \rightarrow \mathbb{R}_+$.

Output: A maximum weight branching B of G .

- ① Set $i := 0$, $G_0 := G$, and $c_0 := c$.
- ② Let B_i be a maximum weight subgraph of G_i with $|\delta_{B_i}^-(v)| \leq 1$ for all $v \in V(B_i)$.
- ③ If B_i contains no circuit **then** set $B := B_i$ and **go to** ⑤.
- ④ Let \mathcal{C} be the set of circuits in B_i . Contract these circuits:
 Let $V(G_{i+1}) := \mathcal{C} \cup (V(G_i) \setminus \bigcup_{C \in \mathcal{C}} V(C))$.
 For $e = (v, w) \in E(G_i)$ let $e' = (v', w')$ and $\Phi_{i+1}(e') := e$, where
 $v' = C$ if $v \in V(C)$ for $C \in \mathcal{C}$, and $v' = v$ if $v \notin \bigcup_{C \in \mathcal{C}} V(C)$, and
 $w' = C$ if $w \in V(C)$ for $C \in \mathcal{C}$, and $w' = w$ if $w \notin \bigcup_{C \in \mathcal{C}} V(C)$.
 Let $E(G_{i+1}) := \{e' = (v', w') : e \in E(G_i), v' \neq w'\}$
 (parallel edges may arise).
 For $e = (v, w) \in E(G_i)$ with $e' = (v', w') \in E(G_{i+1})$ set
 $c_{i+1}(e') := c_i(e)$ if $w' \notin \mathcal{C}$, and
 $c_{i+1}(e') := c_i(e) - c_i(\alpha(e, C)) + c_i(e_C)$ if $w' = C \in \mathcal{C}$, where
 $\alpha(e, C) \in \delta_C^-(w)$ and e_C is some cheapest edge of C .
 Set $i := i + 1$ and **go to** ②.
- ⑤ **While** $i > 0$ **do**:
 Set $B' := (V(G_{i-1}), \{\Phi_i(e) : e \in E(B)\})$.
For each circuit C **of** B_{i-1} **do**:
 If there is an edge $e \in \delta_{B'}^-(V(C))$
then set $E(B') := E(B') \cup (E(C) \setminus \{\alpha(e, C)\})$
else set $E(B') := E(B') \cup (E(C) \setminus \{e_C\})$.
 Set $B := B'$ and $i := i - 1$.