

CAPACITY SCALING ALGORITHM

Input: A digraph G with infinite capacities $u(e) = \infty$ ($e \in E(G)$), numbers $b : V(G) \rightarrow \mathbb{Z}$ with $\sum_{v \in V(G)} b(v) = 0$, and conservative weights $c : E(G) \rightarrow \mathbb{R}$.

Output: A minimum cost b -flow f .

- ① Set $b' := b$ and $f(e) := 0$ for all $e \in E(G)$.
Set $\gamma = 2^{\lfloor \log b_{\max} \rfloor}$, where $b_{\max} = \max\{b(v) : v \in V(G)\}$.
- ② **If $b' = 0$ then stop, else:**
Choose a vertex s with $b'(s) \geq \gamma$.
Choose a vertex t with $b'(t) \leq -\gamma$ such that t is reachable from s in G_f .
If there is no such s or t then go to ⑤.
- ③ Find an s - t -path P in G_f of minimum weight.
- ④ Set $b'(s) := b'(s) - \gamma$ and $b'(t) := b'(t) + \gamma$. Augment f along P by γ .
Go to ②.
- ⑤ **If $\gamma = 1$ then stop.** (There exists no b -flow.)
Else set $\gamma := \frac{\gamma}{2}$ and go to ②.