EDMONDS BRANCHING ALGORITHM

Input: Digraph G, $c : E(G) \to \mathbb{R}_+$

Output: A branching B in G with max weight $c(B) := \sum_{e \in E(B)} c(e)$

① Set $i := 0$, $G_0 := G$, $c_0 := c$

② Construct a subgraph B_i of G_i with max weight and $V(B_i) = V(G_i)$ and $\deg^-_{B_i}(v) \le 1$, $\forall v \in V(B_i)$

③ If B_i cycle-free, then set $B := B_i$ and go to ⑤.

④ Construct (G_{i+1}, c_{i+1}) from (G_i, c_i) by applying the following operations

for every cycle C in B_i do
  Contract C to a single vertex v_C in G_{i+1}
  for every edge (arc) $e = (z, y) \in E(G_i)$ with $z \notin V(C)$ and $y \in V(C)$ do
   • Set $z' := v_{C'}$ if z belong to (another) cycle C' in B_i, and $z' := z$ otherwise
   • Set $e' := (z', v_C)$ and $\phi(e') := e$
   • Set $c_{i+1}(e') := c_i(e) - c_i(\alpha(e, C)) + c_i(e_C)$, where $\alpha(e, C) = (x, y) \in E(C)$ and e_C is the cheapest edge in C
  end for
end for

Set $i = i+1$ and go to ②

⑤ if $i = 0$ then stop; Output B

⑥ for every cycle C in B_{i-1} do
  if $\exists\, e' = (z, v_C) \in E(B)$ then set
   $E(B) = (E(B) \setminus \{e'\}) \cup \phi(e') \cup (E(C) \setminus \{\alpha(\phi(e'), C)\})$
  else set $E(B) := E(B) \cup (E(C) \setminus \{e_C\})$
end for

Set $V(B) := V(G_{i-1})$; $i := i-1$ and go to Step ⑤